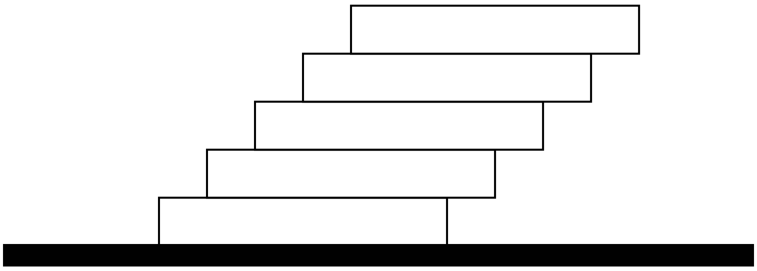
pile of bricks



A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides form continuous planes, but the ends are offset at each brick by a distance L/*a*, where *a* is an integer. How many bricks can be used in this manner before the pile topples over?

Solution by Rudy Arthur:

(All distances are measured from the left end of the bottom brick.)

The centre of mass of the first brick is at L/2; the centre of mass of the second is at $\frac{L}{2}+\frac{L}{a}$, the third is at $\frac{L}{2}+\frac{2L}{a}$, and so on. So the centre of mass of a pile of *n* bricks is at $\sum_{i=1}^{n} m_i r \Big/ \sum_{i=1}^{n} m_i$

$$= \frac{m}{nm}\left(\frac{L}{2}+(\frac{L}{2}+\frac{L}{a})+(\frac{L}{2}+\frac{2L}{a})+...+(\frac{L}{2}+\frac{(n-1)L}{a})\right)$$

$$= \frac{L}{n}\left(\frac{n}{2}+\frac{1}{a}(1+2+...+(n-1))\right)$$

$$= \frac{L}{n}\left(\frac{n}{2}+\frac{1}{2a}n(n-1)\right)$$

$$= \frac{L}{2}\left(1+\frac{(n-1)}{a}\right).$$

The pile will be stable only if its center of mass lies to the left of the right end of the lowest brick, that is, only if $\frac{L}{2}\left(1+\frac{(n-1)}{a}\right) < L$, for which the greatest value *n* can assume is $n = a$. So one can stack at most *a* bricks in this manner.