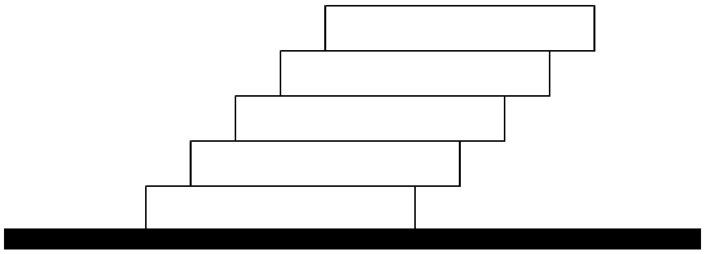
pile of bricks



A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides form continuous planes, but the ends are offset at each brick by a distance L/a, where a is an integer. How many bricks n can be used in this manner before the pile topples over?

<u>Solution by Chris Siegert:</u>

Assume the bricks don't stick together at all. Then the bricks on top of the bottom brick (the "upper bricks") will tip once their center of mass shifts to the right of the bottom brick's <u>top-right-hand</u> corner.

Define:

$x_i = i\frac{L}{a}$, distance from center of bottom brick to center of i^{th} upper brick,

$m =$ mass of a single brick,

$x_{cm} =$ center of mass of the upper bricks,

$n =$ total number of bricks,

$n-1 =$ number of upper bricks,

$M = (n-1)m$, total mass of the upper bricks.

The center of mass of the upper bricks equals the sum of the products of each upper brick's center of mass x_i multiplied by its mass m, divided by the total mass of the upper bricks. So,

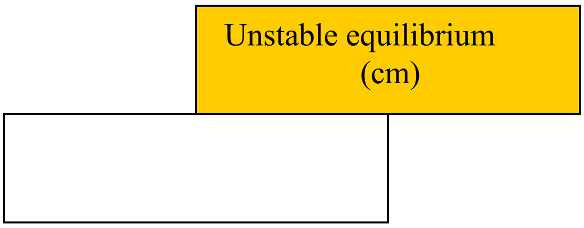
$$x_{cm} = \frac{\sum_{i=1}^{n-1} m x_i}{M} = \frac{\sum_{i=1}^{n-1} m\left(i\frac{L}{a}\right)}{(n-1)m} = \frac{L}{a(n-1)}\sum_{i=1}^{n-1} i = \frac{L}{a(n-1)}\frac{n(n-1)}{2} = \frac{nL}{2a}.$$

The upper bricks will be stable if $x_{cm} < L/2$, in an unstable equilibrium if $x_{cm} = L/2$, and unstable if $x_{cm} > L/2$. Assuming the pile does not topple in the unstable equilibrium condition, we must have $nL/2a \le L/2$, and therefore $n \le a$, so the largest possible n before tipping must be equal to a.

Answer: $n = a.$

Check:

Let a =2 , n =2



Let a = 4 , n = 4