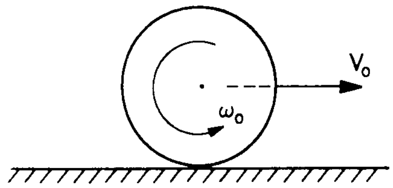
Shooting Marbles



An amusing trick is to press a finger down on a marble, on a horizontal table top, in such a way that the marble is projected along the table with an initial linear speed V0 and an initial backward rotational speed ω0, ω0 being about a horizontal axis perpendicular to V0. The coefficient of sliding friction between marble and top is constant. The marble has radius R.

a) What relationship must hold between Vo, R, and ω0 for the marble to slide to a complete stop?

b) What relationship must hold between Vo, R, and ω0 for the marble to skid to a stop and then start returning toward its initial position, with a final constant linear speed of 3/7 Vo?

Sukumar Chandra's Solution *(using kinematics)*

The kinetic force of friction, $f_k = \mu Mg$, acting horizontally leftward through the point of contact produces leftward translational acceleration μg [Force / mass] and rotational anticlockwise acceleration, $\alpha = 5\ \mu g / 2R$ [torque / I_{cm}] .

a) If the marble comes to a stop after time t, then $V_0 = \mu gt$ [since, $v = u + at$] and $\omega_0 = \alpha t$ [since, $\omega = \omega_0 + \alpha t$]. Eliminating t, we get $V_0 = 2\ \omega_0 R/5$.

b) Final velocity is $- 3V_0 /7$, taking leftward negative. If the marble achieve this position t second after start then, $- 3V_0 /7 = V_0 - \mu gt$. Or,

$$\mu gt = 10V_0 /7. \qquad (1)$$

At this instant it must be rolling with clockwise angular speed $3V_0 /7R$ as $V = \omega R$ must be satisfied for pure rolling. Thus $3V_0 /7R = \omega_0 - \alpha t$ or, $3V_0 /7R = \omega_0 - 5\ \mu gt / 2R$. Or,

$$\mu gt = 2R\omega_0/5 - 6V_0 /35. \qquad (2)$$

From (1) and (2) we get, $10V_0 /7 = 2R\omega_0/5 - 6V_0 /35$, or $V_0 = \omega_0 R/4$.